

AA 6-2 Investigation
Higher degree polynomials

Name _____

We have looked at the graphs of quadratic (2nd degree) and cubic (3rd degree) polynomials. We need to determine how to graph higher degree polynomials (4th, 5th, etc.).

The degree of a polynomial is the exponent of the first term (leading term) if the polynomial was written in standard form.

Complete this chart:

| Standard form polynomial | Leading term | degree |
|-------------------------------|--------------|--------|
| $Y = 3x^2 - 6x - 105$ | $3x^2$ | 2 |
| $Y = -7x^3 + 14x^2 - 9x + 18$ | $-7x^3$ | 3 |
| $Y = x^7 - 4x^6 + 1$ | | |
| $Y = -x^5 + 3x^3 + x^2 - 1$ | | |
| $Y = 4x^8 - 9x + 1$ | | |

If it is in factored form, we must determine what the leading terms would be. As we have seen, the "a" value does not change between forms of the equation. Therefore, we just need to determine the exponent of the first term. This can be as easy as counting x's.

| Factored form polynomial | Leading term | degree |
|--------------------------------|--------------|--------|
| $Y = 3(x-7)(x+5)$ | $3x^2$ | 2 |
| $Y = 5x(x+2)(x+1)$ | $5x^3$ | 3 |
| $Y = -2(x-7)^2(x+3)^3(x-4)$ | $-2x^6$ | 6 |
| $Y = 3(x+4)(x-7)^2(x+3)^5$ | | |
| $Y = x(x-4)(x+8)(x-5)$ | | |
| $Y = -0.2(x+5)^3(x-1)(x+18)^9$ | | |

What did you do to determine the degree and leading term of the polynomial:

Let's explore the graphs of higher degree polynomials. Using Desmos graph each functions and sketch a graph below:

| Equation | Graph | Leading term | Degree | Start | end |
|---------------------------------|-------|--------------|--------|-------|-----|
| $y = x^4 - 21x^2 + 20x$ | | | | | |
| $y = -0.1x(x+4)^3$ | | | | | |
| $y = 0.5(x+3)^2(x+1)(x-1)(x+5)$ | | | | | |

| Equation | Graph | Leading term | Degree | Start | end |
|------------------------------|-------|--------------|--------|-------|-----|
| $y = -0.5x(x-3)^2(x-1)(x+2)$ | | | | | |
| $y = 0.1x^6(x-4)^2$ | | | | | |
| $y = -x^4(x+4)^5$ | | | | | |

In investigation AA6-1 we completed these charts. Are they correct for higher degree polynomials?

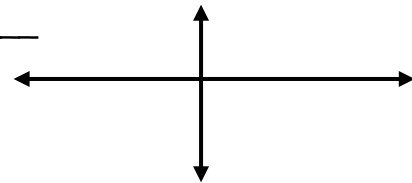
| Factor (multiplicity) | Root | Type (simple, bounce, flat) |
|-------------------------|-------|-----------------------------|
| $(x - 3)$ | (,) | |
| $(x + 3)^{\text{even}}$ | (,) | |
| x^{odd} | (,) | |

| Leading term (first in standard form) | End behavior | |
|---------------------------------------|--------------|-------|
| | Start | Final |
| ax^{even} | up | up |
| ax^{odd} | | |
| $-ax^{\text{even}}$ | | |
| $-ax^{\text{odd}}$ | | |

Based on the graphs you have seen:

What is the maximum number of roots a 3rd degree polynomial can have? _____

Sketch an example.



What is the maximum number of roots an n-degree polynomial can have? _____

For each polynomial function shown below, draw a horizontal line through the graph and state the **minimum** degree the equation could have.

